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Analytical treatment of W-state preparation in cavity QED in the presence of very weak dissipation

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Abstract

We propose an analytical treatment method of generation of multipartite W-type states with a single-resonant interaction of the atoms with the cavity mode in cavity QED, under the consideration of very weak decay from the excited atomic level and from the cavity mode. By manipulating different coupling strengths between the atoms and the cavity mode, we show how to deterministically produce W states, including the standard W state and the one with arbitrary coefficients. The corresponding analytical expressions are given, indicating that the detrimental influence from the atomic spontaneous emission is three times that from the cavity decay. The experimental feasibility of our scheme is also discussed by using current cavity QED techniques.

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It is well known that entanglement is the base of quantum information processing, and the preparation and detection of entangled states have been at the centre of quantum information science. The basic entangled states are generally classified into Bell states [1], GHZ states [2] and W states. Experimentally, the multi-ion GHZ state [3], the three-ion W state [4], and the N-ion W state [5], the three-atom GHZ state [6], and the five-photon GHZ [7] state have been implemented in ion trap, cavity QED and linear optical systems, respectively. The focus of the present paper is on the preparation of W states by cavity QED, because the entanglement of W states is maximally persistent and robust, even under particle loss, among the various kinds of entangled states, and also because no experiment has been reported so far for the generation of the multi-atom W states in cavity QED although some schemes [8–11] have been proposed.

Guo *et al* [8] utilized the atoms interacting with a large-detuning cavity via virtual excitation of the cavity mode to generate a three-atom W state, while the large-detuning results in a rigid constraint for the operational speed, and when the number of the atoms is larger than 3, the scheme cannot deterministically produce W states. Marr *et al* [11] have also studied the multi-atom W states by means of decoherent-free subspace and adiabatic passages.

In this work, we propose an analytical treatment method of generation of multipartite W states by a single-resonant interaction of the atoms with the cavity mode in cavity QED. The scheme only requires the atoms going through the cavity simultaneously and resonantly interacting with the cavity mode, without the need of any additional classical fields. By manipulating different coupling strengths between different atoms and the cavity mode, the W state can be deterministically generated, including the standard W state and that with arbitrary coefficients. We will first treat analytically an ideal situation and then turn to the analytical treatment of a real case with the cavity decay and atomic spontaneous emission involved. Our scheme is suitable for implementation in both microwave and optical cavities and is close to the reach with current cavity QED techniques.

First of all, we consider a resonant interaction of N two-level atoms with a single-mode cavity, where the employed internal states of the atom j are the groundstate $|g_j\rangle$ and the excited state $|e_j\rangle$. So the Hamiltonian in units of $\hbar = 1$ is

$$H_i = \sum_{j=1}^{N} g_{jc}\left(a^+ S_j^- + a S_j^+\right), \tag{1}$$

where g_{jc} is the coupling constant of the jth atom to the cavity mode and $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$ are the atomic spin operators for raising and lowering, respectively. a^+(a) is the creation (annihilation) operator for the cavity mode. Assuming that the atoms and the cavity mode are initially in the state $\prod_{j=2}^{N}|e_1\rangle|g_j\rangle$ and in the vacuum state $|0\rangle$, respectively, we have the evolution of the system,

$$\begin{aligned}
|\psi(t)\rangle &= U(t)\prod_{j=2}^{N}|e_1\rangle|g_j\rangle|0\rangle \\
&= \exp\left[-\mathrm{i}t\sum_{j=1}^{N} g_j\left(a^+ S_j^- + a S_j^+\right)\right]\prod_{j=2}^{N}|e_1\rangle|g_j\rangle|0\rangle \\
&= \left[\left(g_1^2/G^2\right)\cos(Gt) + \left(G^2 - g_1^2\right)/G^2\right]\prod_{j=2}^{N}|e_1\rangle|g_j\rangle|0\rangle \\
&\quad + (g_1/G^2)\{\cos(Gt) - 1\}\sum_{k=2}^{N} g_k|g_1\rangle|e_k\rangle \prod_{j=2,\, j\neq k}^{N}|g_j\rangle|0\rangle \\
&\quad - \mathrm{i}(g_1/G)\sin(Gt)\prod_{j=1}^{N}|g_j\rangle|1\rangle,
\end{aligned} \tag{2}$$

with $G = \sqrt{\sum_{j=1}^{N} g_{jc}^2}$. We will construct W states based on equation (2). In the case of $Gt = \pi$ and $g_{1c}^2 = G^2 - g_{1c}^2 = \sum_{j=2}^{N} g_{jc}^2$, we acquire from equation (2)

$$|\psi(t)\rangle = -(1/g_{1c})\sum_{k=2}^{N} g_k|e_k\rangle \prod_{j=2,\, j\neq k}^{N}|g_j\rangle|g_1\rangle|0\rangle. \tag{3}$$

So a W state with arbitrary coefficients is deterministically generated, i.e.,

$$W_{N-1}^{P} = (1/g_{1c})\sum_{k=2}^{N} g_{kc}|e_k\rangle \prod_{j=2, j\neq k}^{N} |g_j\rangle, \tag{4}$$

in which the states of the first atom and the cavity mode are dropped and the interaction time is $t = \frac{\pi}{\sqrt{2}g_{1c}}$. If the coupling strengths of other atoms, except the first atom, to the cavity mode are equal, e.g., $g_{kc} = g_{1c}/\sqrt{N-1}$, $(k = 2, \ldots, N)$, which can be realized through controlling positions of different atoms in the cavity, we may produce a standard W state:

$$\begin{aligned} W_{N-1}^{S} &= \frac{1}{\sqrt{N-1}}\sum_{k=2}^{N}|e_k\rangle \prod_{j=2, j\neq k}^{N} |g_j\rangle \\ &= \frac{1}{\sqrt{N-1}}[|e_2\rangle|g_3\rangle\cdots|g_N\rangle + |g_2\rangle|e_3\rangle|g_4\rangle\cdots|g_N\rangle + \cdots + |g_2\rangle|e_3\rangle\cdots|g_{N-1}\rangle|e_N\rangle]. \end{aligned} \tag{5}$$

We now investigate the influences from the very weak cavity decay and the atomic spontaneous emission on the generation of above W states. Under the condition of a single-photon interaction of atoms with the cavity mode, consider the following Hamiltonian [12]:

$$H_d = \sum_{j=1}^{N} g_{jc}\left(a^{+}S_j^{-} + aS_j^{+}\right) - \mathrm{i}\frac{\kappa}{2}a^{+}a - \mathrm{i}\frac{\Gamma}{2}\sum_{j=1}^{N}|e_j\rangle\langle e_j|, \tag{6}$$

where κ and Γ are the cavity decay and the atomic spontaneous emission rates, respectively. We here consider very weak decay effect to make sure that neither photon decay nor spontaneous emission really occur during the implementation period of our scheme. Provided that the atoms and the cavity mode are initially in the state $\prod_{j=2}^{N}|e_1\rangle|g_j\rangle$ and in the vacuum state $|0\rangle$, respectively, we can obtain, by straightforward algebra, the evolution of the system before the leakage of photons from the cavity,

$$\begin{aligned} |\psi_{\mathrm{decay}}(t)\rangle &= U_d(t)\prod_{j=2}^{N}|e_1\rangle|g_j\rangle|0\rangle \\ &= \exp\left\{-\mathrm{i}t\left[\sum_{j=1}^{N} g_{jc}\left(a^{+}S_j^{-} + aS_j^{+}\right) - \mathrm{i}\frac{\kappa}{2}a^{+}a - \mathrm{i}\frac{\Gamma}{2}\sum_{j=1}^{N}|e_j\rangle\langle e_j|\right]\right\}\prod_{j=2}^{N}|e_1\rangle|g_j\rangle|0\rangle \\ &= \left\{\left(g_{1c}^2/G^2\right)\exp[-(\kappa+\Gamma)t/4]\left[\cos(A_\kappa t) + \frac{\kappa-\Gamma}{4A_\kappa}\sin(A_\kappa t)\right]\right. \\ &\quad \left. + \left[\left(G^2 - g_{1c}^2\right)/G^2\right]\exp(-\Gamma t/2)\right\}\prod_{j=2}^{N}|e_1\rangle|g_j\rangle|0\rangle \\ &\quad + \left(g_{1c}/G^2\right)\left\{\exp[-(\kappa+\Gamma)t/4]\left[\cos(A_\kappa t) + \frac{\kappa-\Gamma}{4A_\kappa}\sin(A_\kappa t)\right] - \exp(-\Gamma t/2)\right\} \\ &\quad \times \sum_{k=2}^{N} g_{kc}|g_1\rangle|e_k\rangle \prod_{j=2, j\neq k}^{N}|g_j\rangle|0\rangle - (\mathrm{i}g_{1c}/A_\kappa)\sin(A_\kappa t)\prod_{j=1}^{N}|g_j\rangle|1\rangle, \end{aligned} \tag{7}$$

with $G = \sqrt{\sum_{j=1}^{N} g_{jc}^2}$ and $A_\kappa = \sqrt{\sum_{j=1}^{N} g_{jc}^2 - (\kappa-\Gamma)^2/16}$. By choosing

$$A_\kappa t = \pi, \tag{8}$$

and

$$g_{1c}^2\big/\big(G^2 - g_{1c}^2\big) = \exp[(\kappa - \Gamma)t/4], \tag{9}$$

we have the evolution of the system

$$|\psi_{\text{decay}}(t)\rangle = -\frac{\exp(-\Gamma t/2)}{g_{1c}} \times \sum_{k=2}^{N} g_{kc}|e_k\rangle \prod_{j=2,\, j\neq k}^{N} |g_j\rangle|g_1\rangle|0\rangle, \tag{10}$$

which implies a W state of $(N-1)$ atoms with arbitrary coefficients and with dissipative effects involved, i.e.,

$$\begin{aligned} W_{N-1}^{P} &= (1/g_{1c})\exp(-\Gamma t/2)\sum_{k=2}^{N} g_{kc}|e_k\rangle \prod_{j=2,\, j\neq k}^{N}|g_j\rangle \\ &= (1/g_{1c})\exp(-\Gamma t/2)[g_{2c}|e_2\rangle|g_3\rangle\cdots|g_N\rangle + g_{3c}|g_2\rangle|e_3\rangle|g_4\rangle\cdots|g_N\rangle \\ &\quad + \cdots + g_{Nc}|g_2\rangle|g_3\rangle\cdots|g_{N-1}\rangle|e_N\rangle]. \end{aligned} \tag{11}$$

If the coupling strengths of other atoms, except the first one, to the cavity mode are equal, e.g.,

$$g_{kc} = \frac{g_{1c}}{\sqrt{N-1}} \exp\left[\frac{(\Gamma-\kappa)t}{8}\right], \tag{12}$$

which is from equation (9), we can produce a standard W state of $N-1$ atoms:

$$\begin{aligned} W_{N-1}^{S} &= \frac{1}{\sqrt{N-1}}\exp[-(3\Gamma+\kappa)t/8] \times \sum_{k=2}^{N}|e_k\rangle \prod_{j=2,\, j\neq k}^{N}|g_j\rangle \\ &= \frac{1}{\sqrt{N-1}}\exp[-(3\Gamma+\kappa)t/8] \times [|e_2\rangle|g_3\rangle\cdots|g_N\rangle \\ &\quad + |g_2\rangle|e_3\rangle|g_4\rangle\cdots|g_N\rangle + \cdots + |g_2\rangle|g_3\rangle\cdots|g_{N-1}\rangle|e_N\rangle]. \end{aligned} \tag{13}$$

It is specially noted that both the cavity decay and the atomic spontaneous emission result only in a global damping factor in equation (13) without any influence on the internal structure of the W state. This originates from the special choice of the couplings, i.e., the symmetry of the atomic space distribution via the cavity mode in the cavity. Although dissipation can destroy entanglement, the internal entanglement structure remains unchanged in the process. When there is no leakage of photons out of the cavity during the evolution, we have the success probability

$$P = \exp[-(3\Gamma+\kappa)t/4], \tag{14}$$

where the detrimental influence from the atomic spontaneous emission is three times that from the cavity decay. From equations (8) and (9), we have the interaction time determined by

$$\sqrt{g_{1c}^2\{1+\exp[(\Gamma-\kappa)t/4]\} - (\Gamma-\kappa)^2/16}\,t = \pi, \tag{15}$$

which is independent of the atomic number. In particular, when $\Gamma = \kappa$, the resulting interaction time is the same as the ideal case $t = \pi/\sqrt{2}g_{1c}$, and thus the success probability reduces to $P = \exp[-\kappa\pi/(\sqrt{2}g_{1c})]$.

We briefly discuss the experimental feasibility of our proposed scheme. To achieve a standard multipartite W state, we require that the first atom coupling to the cavity be different from other $(N-1)$ atoms, while the other $(N-1)$ atoms should have identical coupling strengths to the cavity mode. We usually employ the relation $g_c = g_{0c}\cos\left(\frac{2\pi z}{\lambda_0}\right)\exp(-r^2/w^2)$

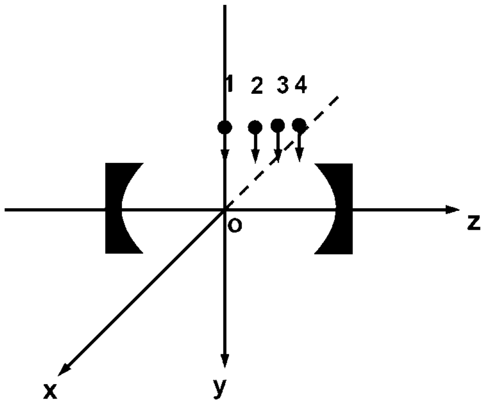

Figure 1. Schematics of our scheme by sending, for example, four atoms through a cavity along y-axis. Except the first atom, the other three atoms experience identical coupling to the standing wave of the cavity mode.

[13, 14] to describe the coupling strengths between the atoms and the cavity, where g_{0c} is the coupling strength at the cavity centre, $r(=\sqrt{x^2+y^2})$ is the distance of the atom from the cavity centre, and λ_0 and w are, respectively, the wavelength and the waist of the cavity mode. For the clarity and convenience of our discussion, we will only consider below the ideal case in our estimate of the parameter numbers. The extension to the dissipative situation is easy and straightforward by using equations (8) to (12). In the case of the microwave cavity, we suppose $g_c \simeq g_{0c}\cos\left(\frac{2\pi z}{\lambda_0}\right)$ [13, 14], and thereby acquire the standard W state if the atomic positions satisfy the conditions $z_1 = 0$ and $z_i = \frac{\lambda_0}{2\pi}\left[\arccos\left(\frac{1}{\sqrt{N-1}}\right) \pm (i-2)\pi\right]$, $i = 2, \ldots, N$, as shown in figure 1.

This method might also be applied to optical cavity QED if $g_c \simeq g_{0c}\cos\left(\frac{2\pi z}{\lambda_0}\right)$ is also satisfied there. We have noticed coherent interaction between two atoms in a recent experiment with a microwave cavity [14] and some significant advances for the strong coupling of the atoms to the cavity mode and for the identification of individual atoms in optical cavity QED [13, 15].

Alternatively, we may consider the circular structure of atoms in an optical cavity, as shown in figure 2. Since the atoms are located in the x–y plane, we have $g_c = g_{0c}\exp(-r^2/w^2)$, and the requirement $g_{kc}(k = 2, \ldots, N) = g_{ic}$ and $g_{1c}^2 = \sum_{j=2}^{N} g_{jc}^2$ can be satisfied if the first atom is located at the centre of cavity ($g_{1c} = g_{0c}$) and other atoms at a circle with the radius $r_k = w\sqrt{\ln(N-1)}$ on the x–y plane. In this case, we can generate the standard W state in equation (5). In a more general situation, provided that the first atom is located at the centre of cavity ($g_{1c} = g_{0c}$) and other atoms at a circle with $r_k = w\sqrt{\ln(g_{0c}/g_{kc})}$ $(k = 2, \ldots, N)$, and with the condition $g_{1c}^2 = \sum_{j=2}^{N} g_{jc}^2$ being satisfied, we can obtain the W state in equation (4). To achieve this method, however, we have to fix the atoms in the optical cavity, which can be done by optical lattices. We have found an experimental report to transport atoms into an optical cavity with optical lattices [16]. But to satisfy our requirement with each atoms confined in a certain lattice, respectively, we expect a more advanced technique in this respect.

The interaction time $t[= \pi/(\sqrt{2}g_{0c})]$ is about 1.1×10^{-2} μs in the optical cavity due to $g_{0c} = 2\pi \times 34$ MHz [13], which is shorter than both the decay time 0.38 μs of the atomic excited state and the cavity decay time 0.24 μs [13]. In a microwave cavity, this interaction time is about 7.1 μs due to $g_{0c} = 2\pi \times 49$ kHz [14], which is much shorter than the cavity

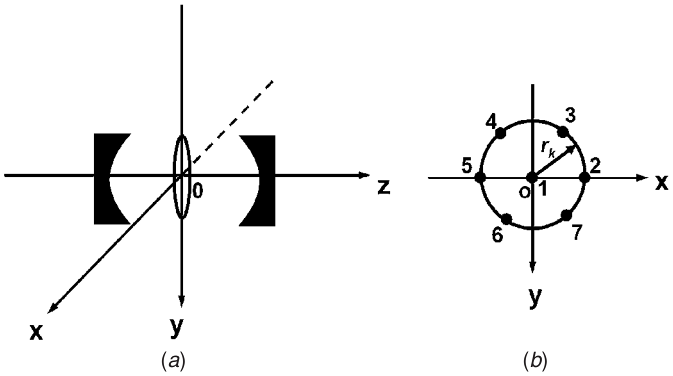

Figure 2. (*a*) The schematics of our scheme by an optical cavity, where the N atoms are located in the x–y plane. (*b*) Spatial distribution of seven atoms in the x–y plane, where the first atom is located at the centre and others are located at the circle with the radius $r_k = w\sqrt{\ln(N-1)}$.

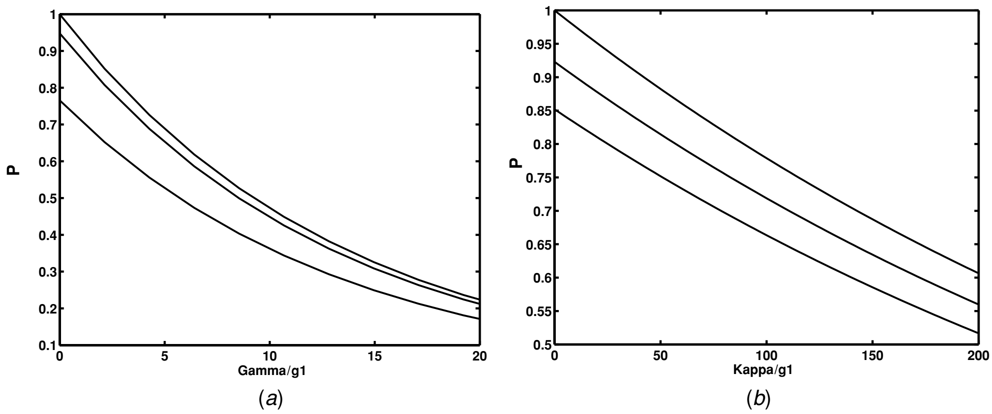

Figure 3. (*a*) The success probability P of the W-state generation versus the atomic decay rate Γ/g_{1c} by equations (14) and (15), where the curves from top to bottom correspond to the cavity decay rates $\kappa = 0, g_{1c}/100, g_{1c}/20$, respectively. (*b*) The success probability P of the W-state generation versus the relative cavity decay κ/g_{1c} by equations (14) and (15), where the curves from top to bottom correspond to the atomic decay rates $\Gamma = 0, g_{1c}/200, g_{1c}/100$, respectively.

decay time 1 ms and the atomic lifetime 30 ms [14]. We numerically present the success rate of our scheme for different cases in figure 3.

Before finishing the paper, we will have specifically discussed equation (13). Since the dissipative rates regarding the cavity mode and the excited atomic level are supposed to be much smaller than the Rabi frequencies g_{ic}, it is not surprising to have the detrimental effect on the entangled state only globally. Once the dissipation really occurs during our implementation time, the internal structure of the prepared entangled state will be definitely affected, and in this case the problem has to be resorted to the master equation by density matrix operators or the Caldeira–Leggett quantum master equation [17]. Therefore our solution based on wavefunction evolution is only valid under the condition with $\mathrm{Max}\{\Gamma, \kappa\} \ll \mathrm{Min}\{g_{ic}\}$. Under this condition, we have analytically demonstrated the detrimental effect of the decays on the prepared entangled states. Particularly, we have quantitatively shown the influence from the spontaneous emission to be stronger than that from the cavity decay by three times. In fact, our method is different from that of the Cadeira–Leggett model [17], where the effect of damping on the quantum interference of two Gaussian wave packets in a harmonic potential and the influence of dissipation on quantum tunnelling in the macroscopic system were calculated and investigated by the Feynman–Vernon influence-functional technique [18]. Recently, this method was applied to the solution of Brownian motion in periodic potentials by Garcia–Palacios and Zueco [19]. On the other hand, the phenomenological operator approach to dissipation in cavity electrodynamics has been proposed in [20], where equation (5) in our scheme can be derived by means of some approximate conditions. Since the above condition [17–19] of our scheme has already been met in the current microwave cavity system, and will, we believe, be met in future in the optical cavity system, our analytical solution would be very helpful for understanding the results from related experiments.

In conclusion, we have proposed the analytical treatment of the W-state preparation in the presence of very weak dissipation including the cavity decay and the atomic spontaneous emission and explored the experimental feasibility of its realization with a single-resonant interaction of the atoms with the cavity mode in cavity QED. It is concluded that although the dissipation can destroy the entanglement, the internal entanglement structure remains unchanged in the process, and that the detrimental influence from the atomic spontaneous emission is three times that from the cavity decay. Since it involves the dissipations from the atomic excited level and the cavity mode, our treatment is closer to a real case in laboratory than any previous scheme [8–10].

Acknowledgments

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